Equation of forced Oscillations.

Let us consider a system oscillating about an equilibrium position under an external periodic force. Let x be its instantaneous displacement for the equilibrium position. Its instantaneous velocity is $\frac{dx}{dt}$, then forces acting upon the system at instant are —

(i) A restoring force proportional to the displacement but acting in opposite direction. This may be written as :— $-Sx$.

where S is the force constant per unit displacement.

(ii) A frictional force proportional to the velocity, but acting in the opposite direction. This may be written as :— $-k\frac{dx}{dt}$

where k is a positive constant per unit velocity.

(iii) An external periodic force is represented by $f_0 \sin pt$. where f_0 is the maximum value of this force and p is its angular velocity.

Thus the total force F acting upon the system is

$$F = -Sx - k\frac{dx}{dt} + f_0 \sin pt \longrightarrow (1)$$

By Newtons second law of motion, this force must be equal to the product of the mass m of the system and the instantaneous acceleration is $\frac{d^2x}{dt^2}$.

$$\therefore F = -Sx - k\frac{dx}{dt} + f_0 \sin pt = m\frac{d^2x}{dt^2}$$

(P.T.O)